Visual Computations, Geometry, and Visual Cortex

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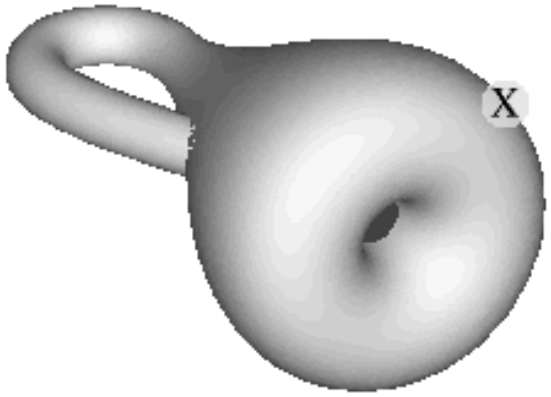
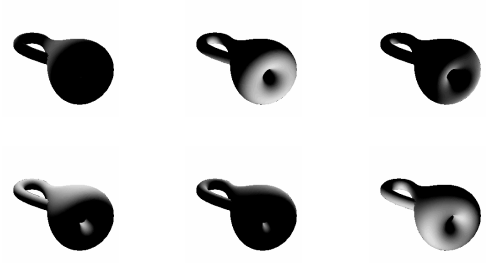
The Motivation for Edge Detection

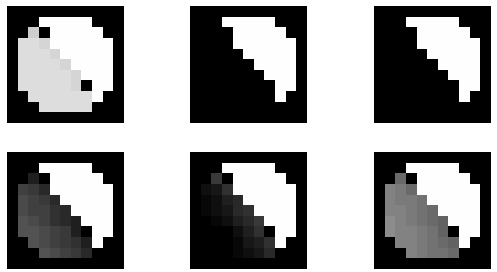
Edges are considered to be stable features of objects in images, *e.g.*, with respect to variable illumination.



An image of a Klein bottle.



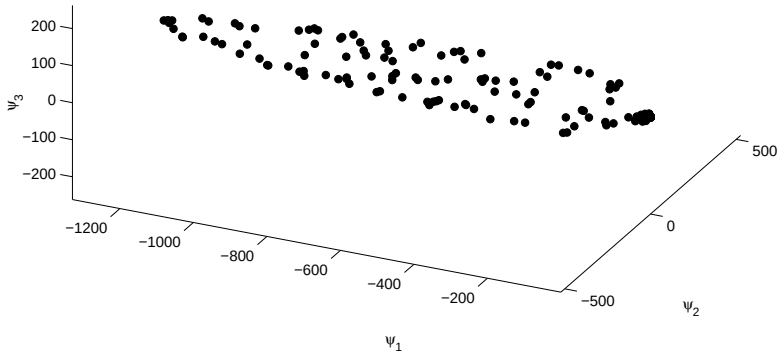
The same object illuminated differently.

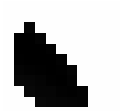


The patch X under different illuminations.

The Motivation for Edge Detection

In image space, the edge patches lie near a low-dimensional manifold.



ψ_1 

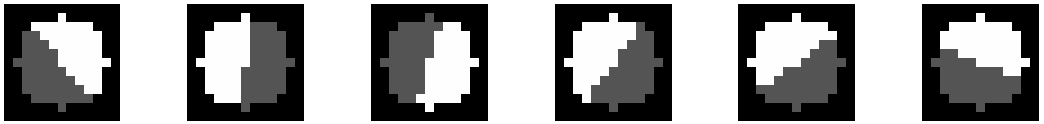
ψ_2 

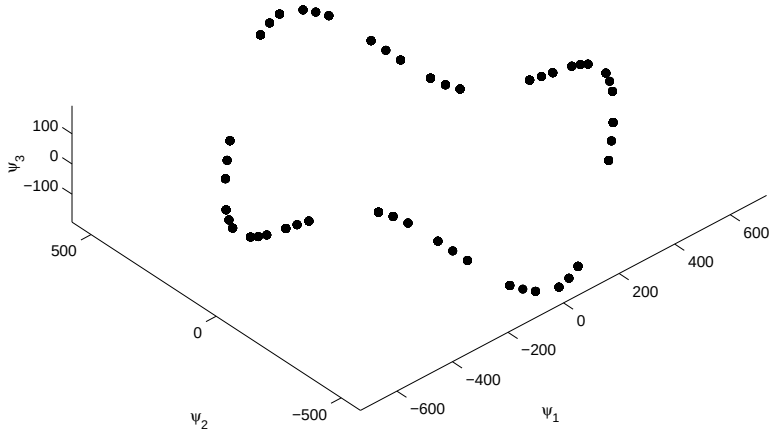
ψ_3 

The Edge Manifold

A simple edge model:

$$f(x, y; \theta) = \begin{cases} 1 & \text{if } -\sin(\theta)x + \cos(\theta)y > 0 \\ 0 & \text{otherwise} \end{cases} .$$

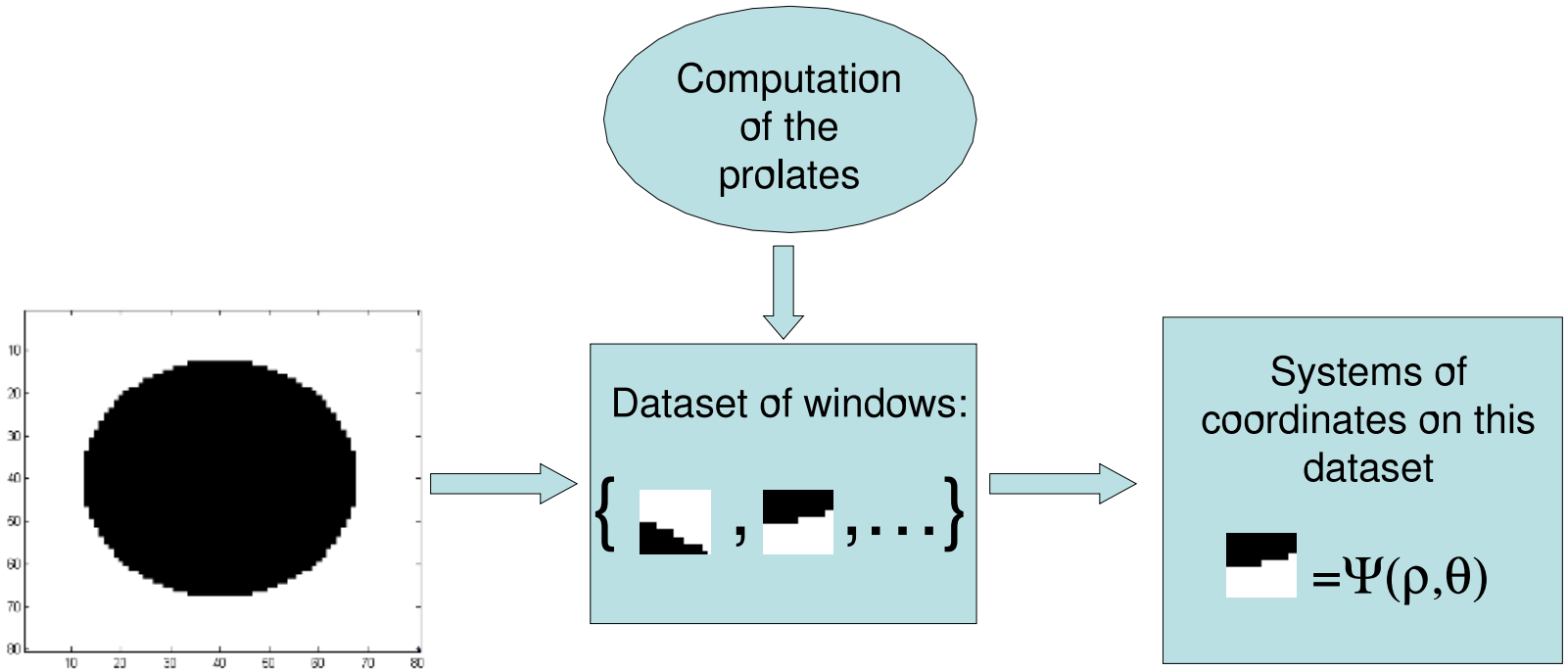




ψ_1 

ψ_2 

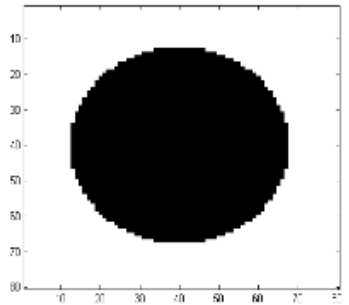
ψ_3 

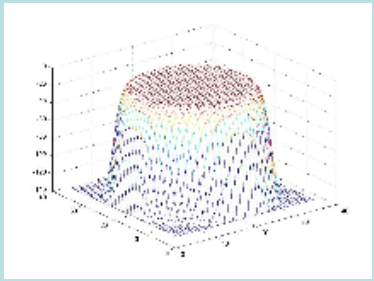
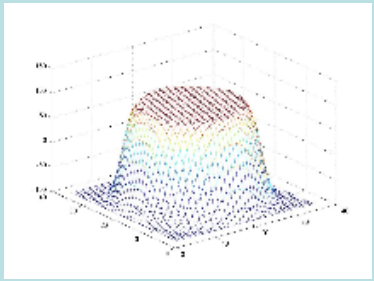
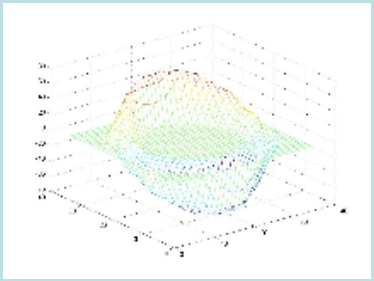
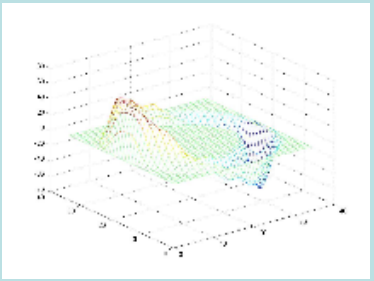


Given an image:

2. Split this image into local patches: to each pixel of the image, associate its 8x8 pixel neighborhood.
3. Treat these windows as a dataset of points in 64 dimensions. Compute the prolates on this set.
4. Find local coordinate systems on the set.

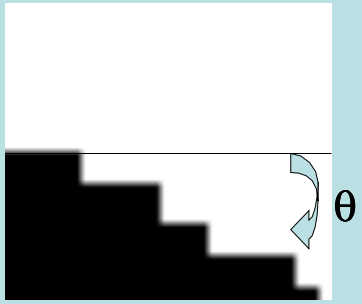
A simple example: black disk on white background:



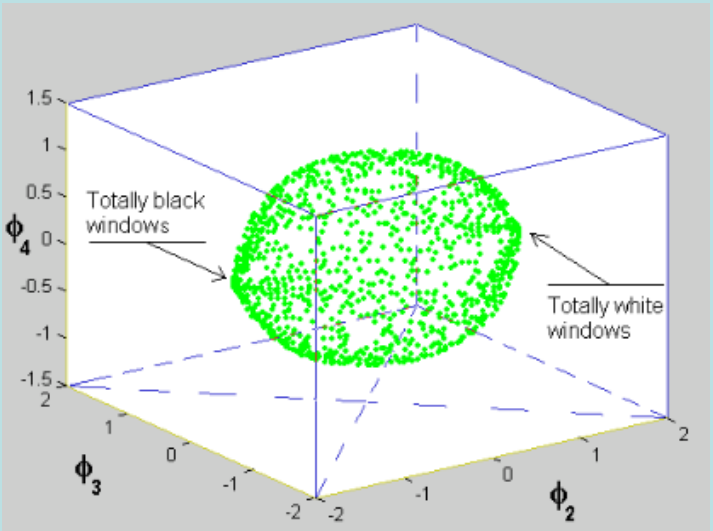





Above are represented the first 4 prolates in the image space (image domain vs. prolate value).

2. Prolates 1 and 2 capture the ratio of black pixels over white pixels.
3. Prolates 3 and 4 capture the angle θ
4. Locally, 2 prolates are sufficient to describe the data

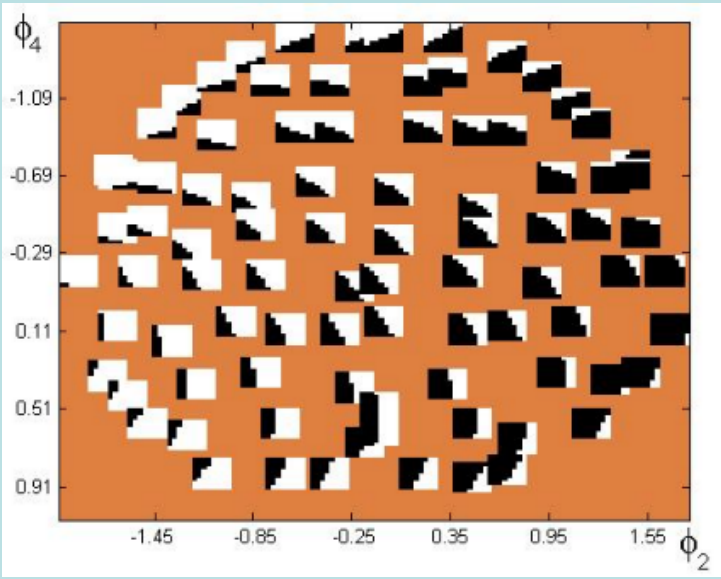


To each point in the dataset (or to each pixel in the image) we associate its coordinates in the (phi2,phi3,phi4) system.

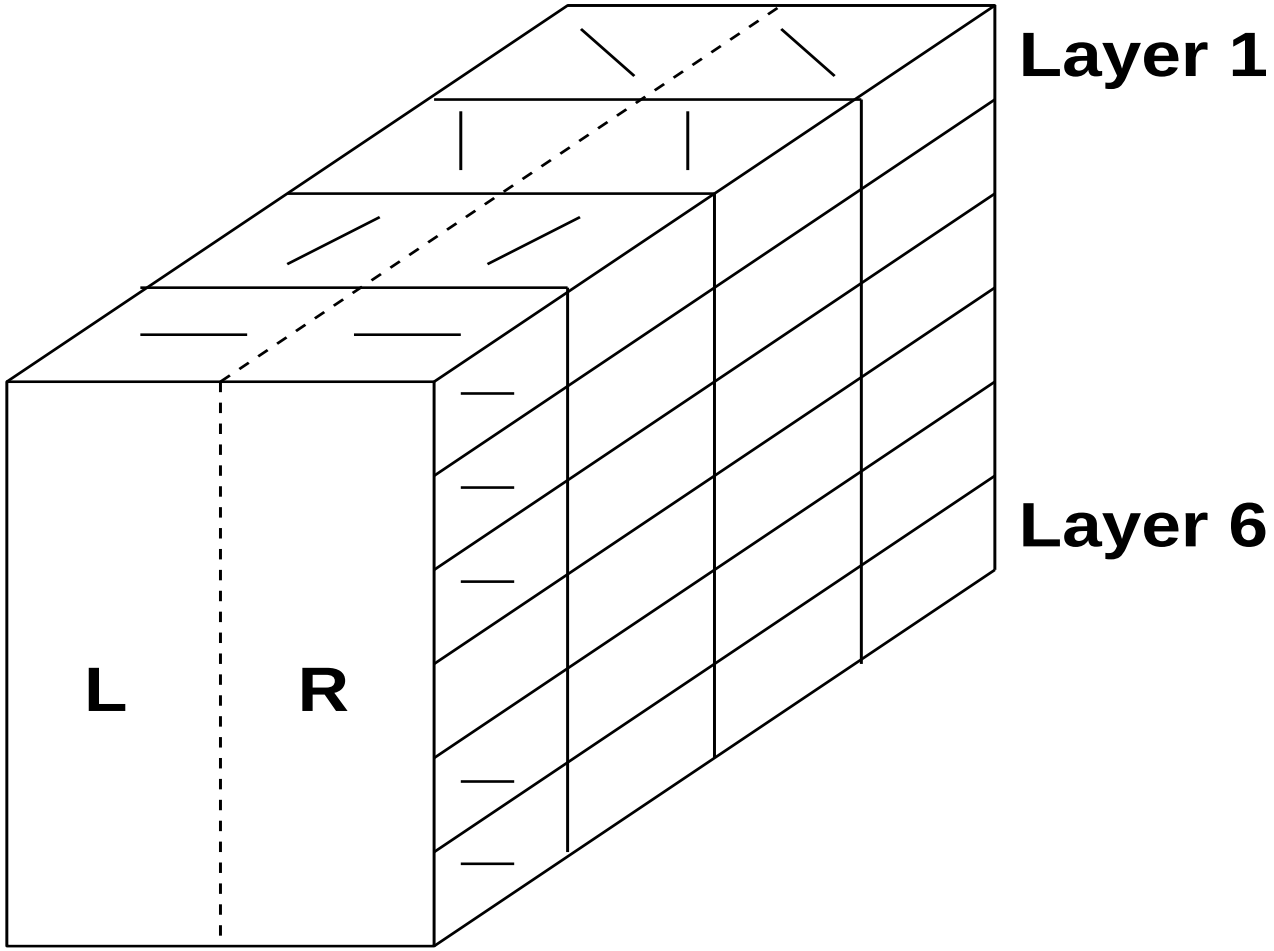


Plot of a selected subset of windows in (phi2,phi4)

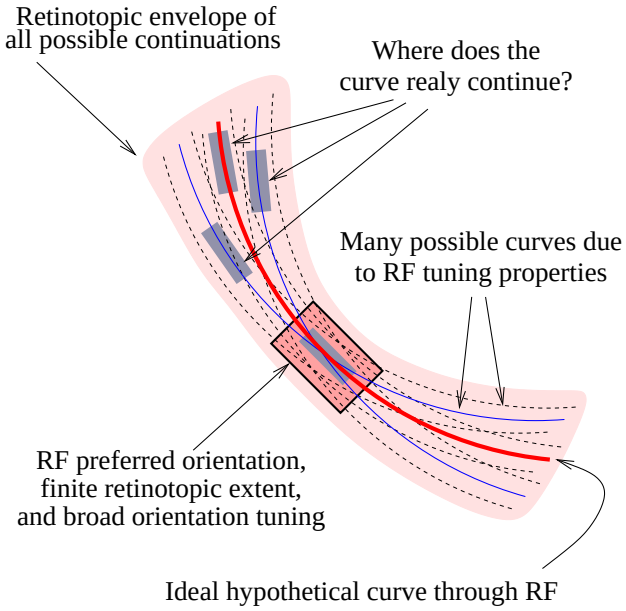
- Phi2 measures the proportion of black pixels
- Phi4 measures the orientation of the edge

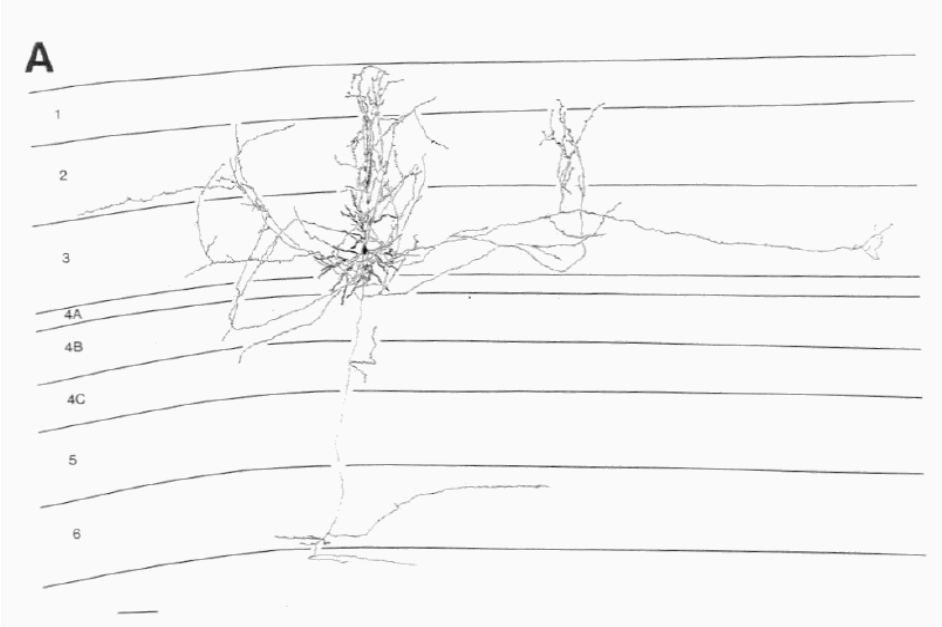


"Ice cube" model of cortical columns



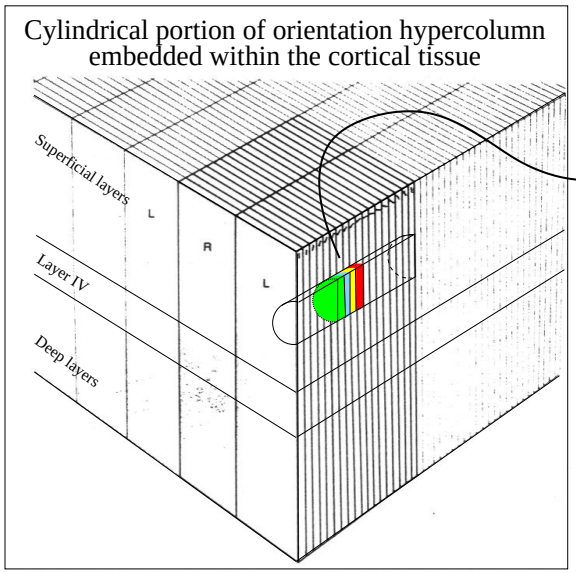
Curve coherence: computational and biological perspectives

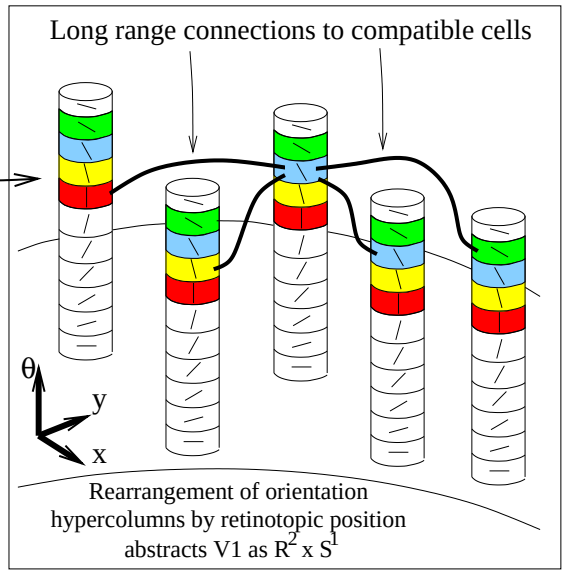


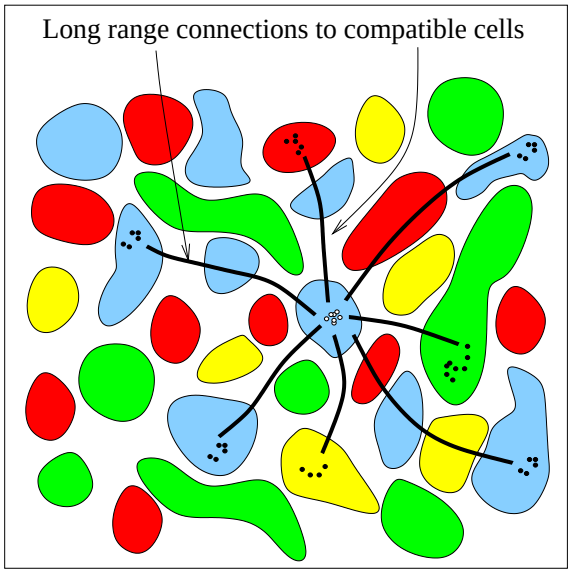


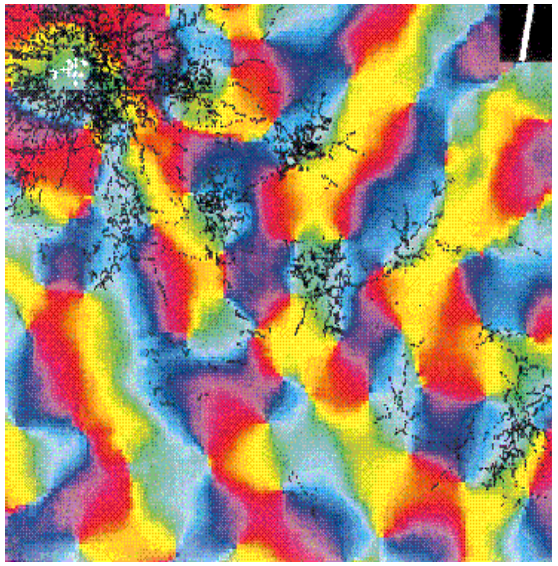
Gilbert, Neuron 1989

Gilbert & Wiesel, J. of Neuroscience, 1983

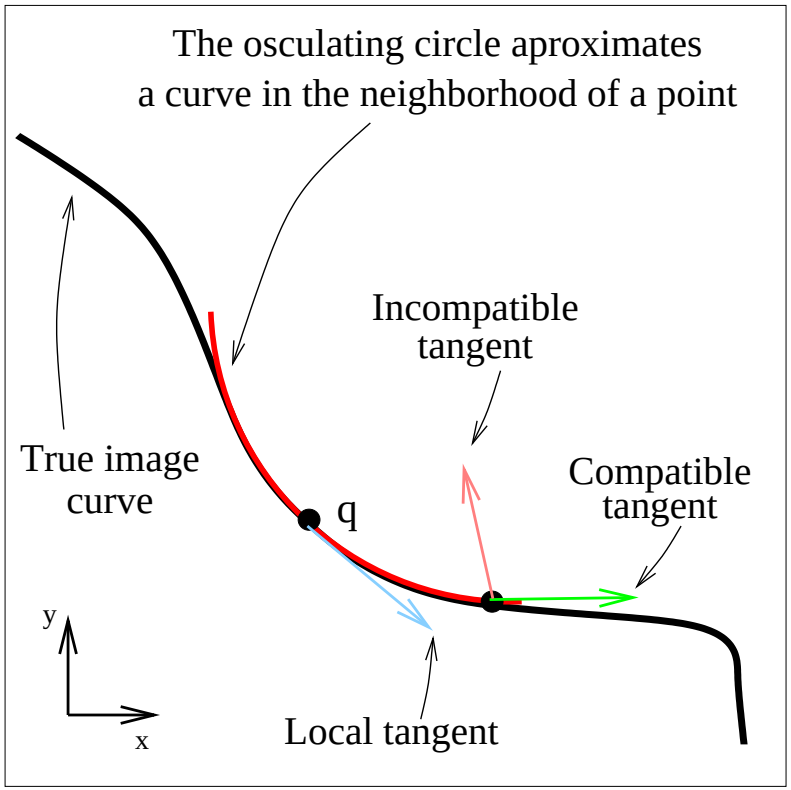


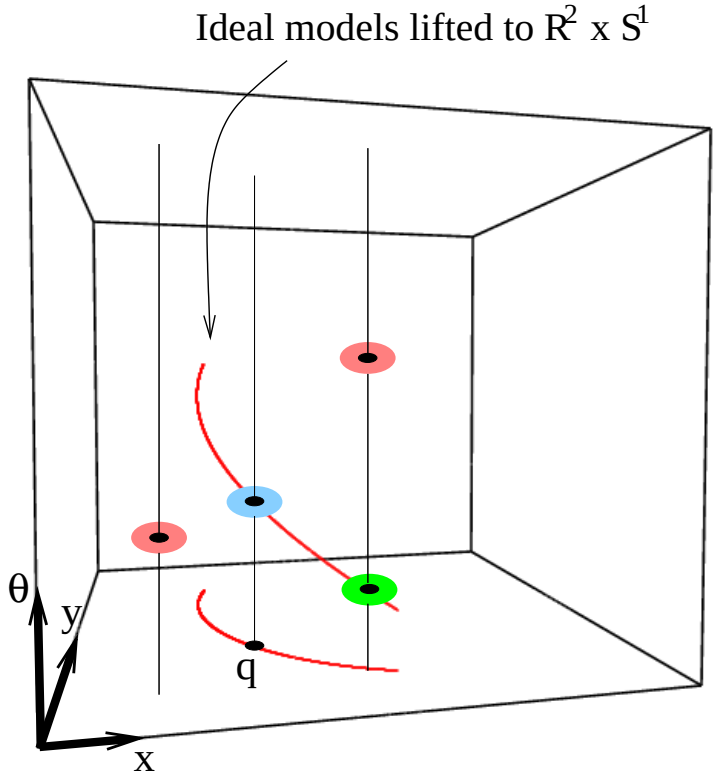




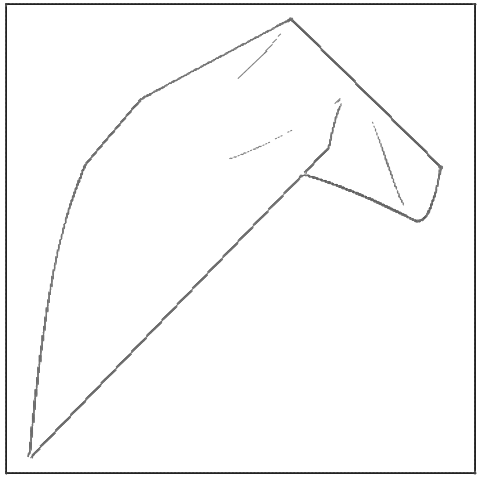


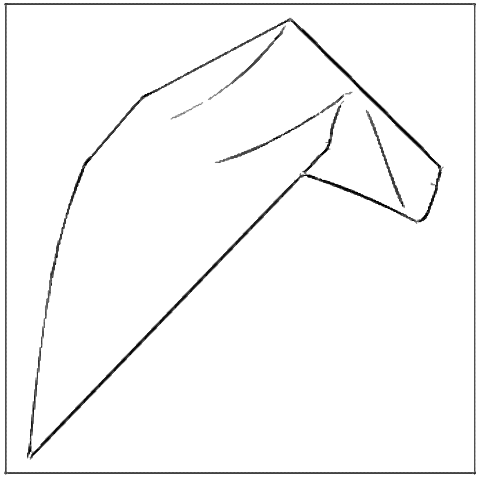
Cocircularity coherence in the orientation hypercolumn $(XY\theta)$ architecture

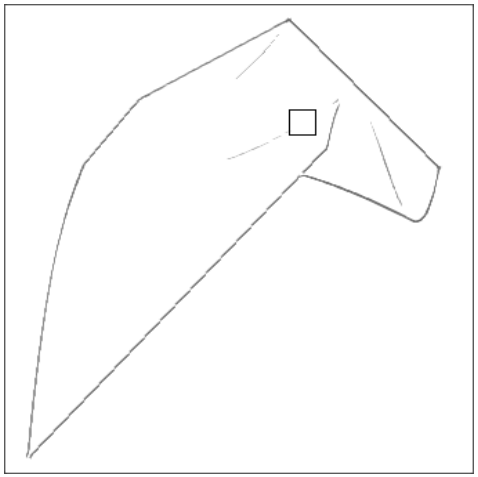


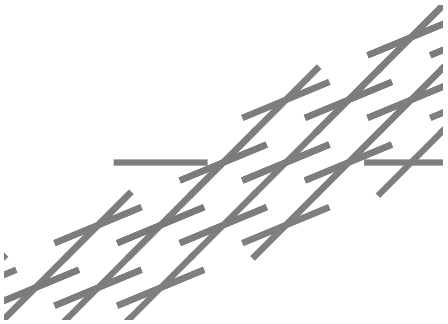


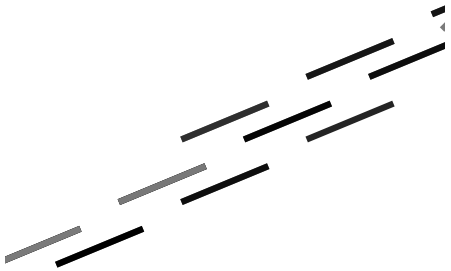
Cocircularity-based curve inference



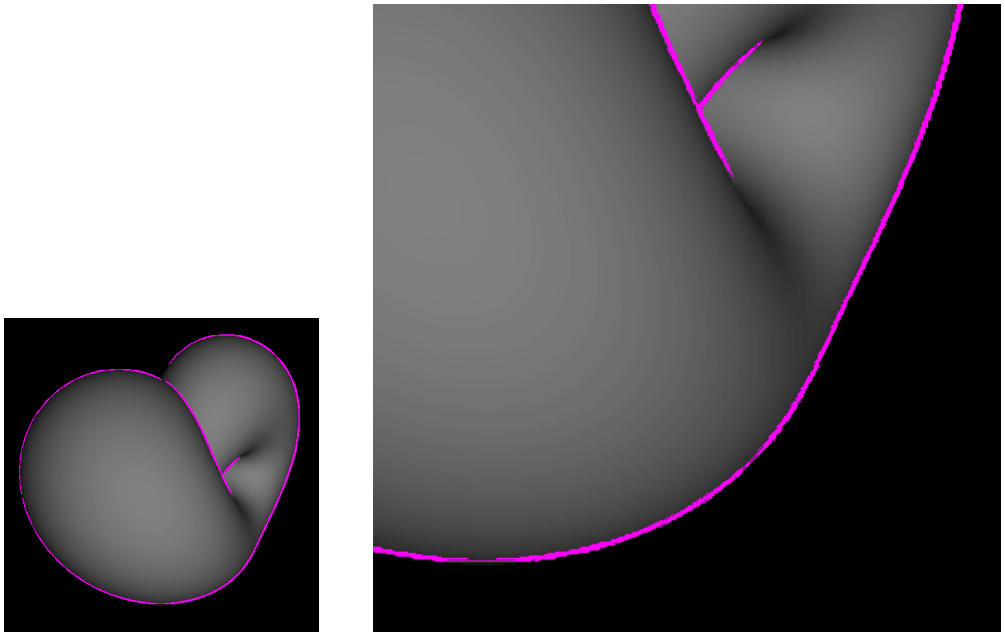




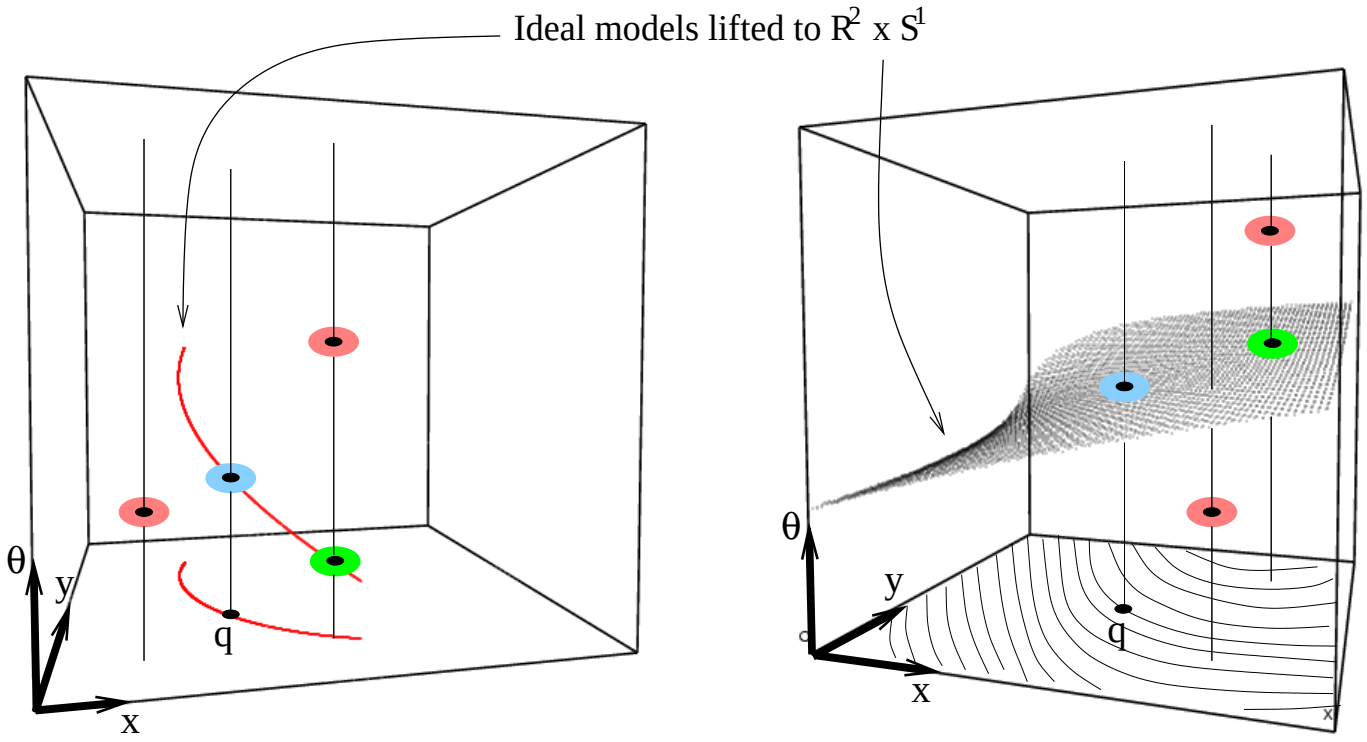




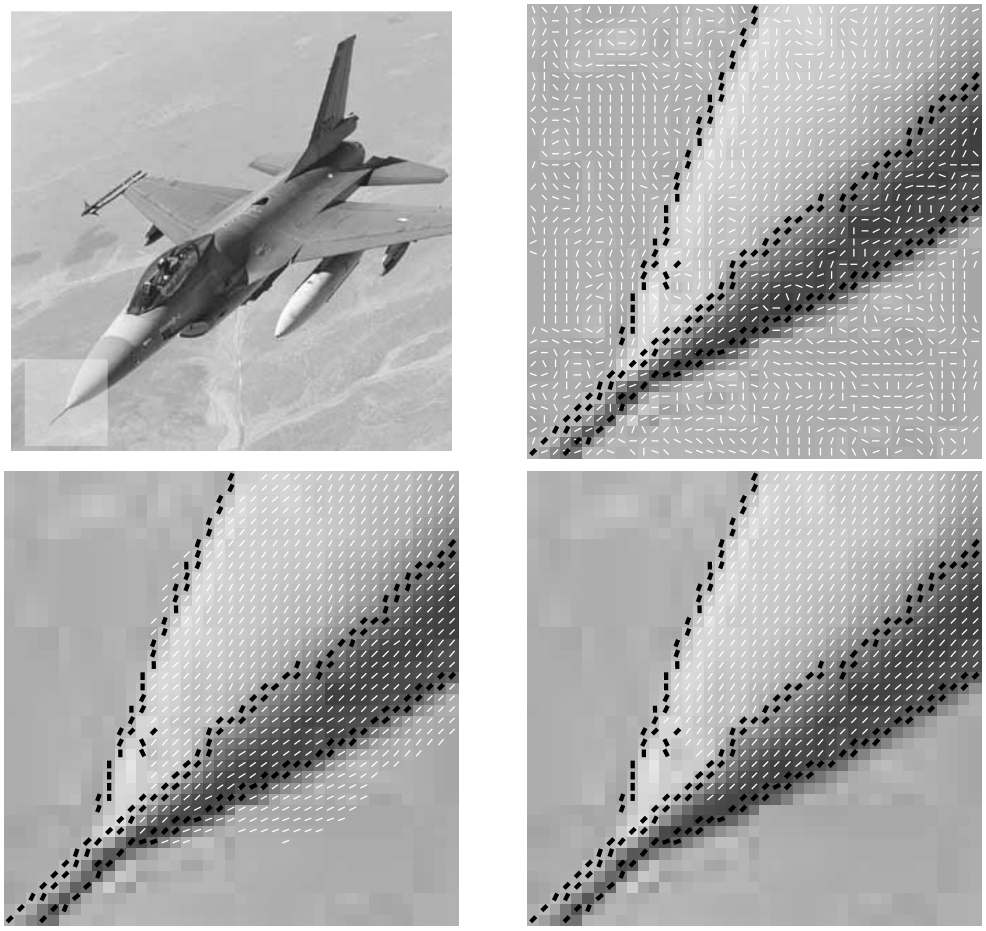
Cocircularity-based curve inference

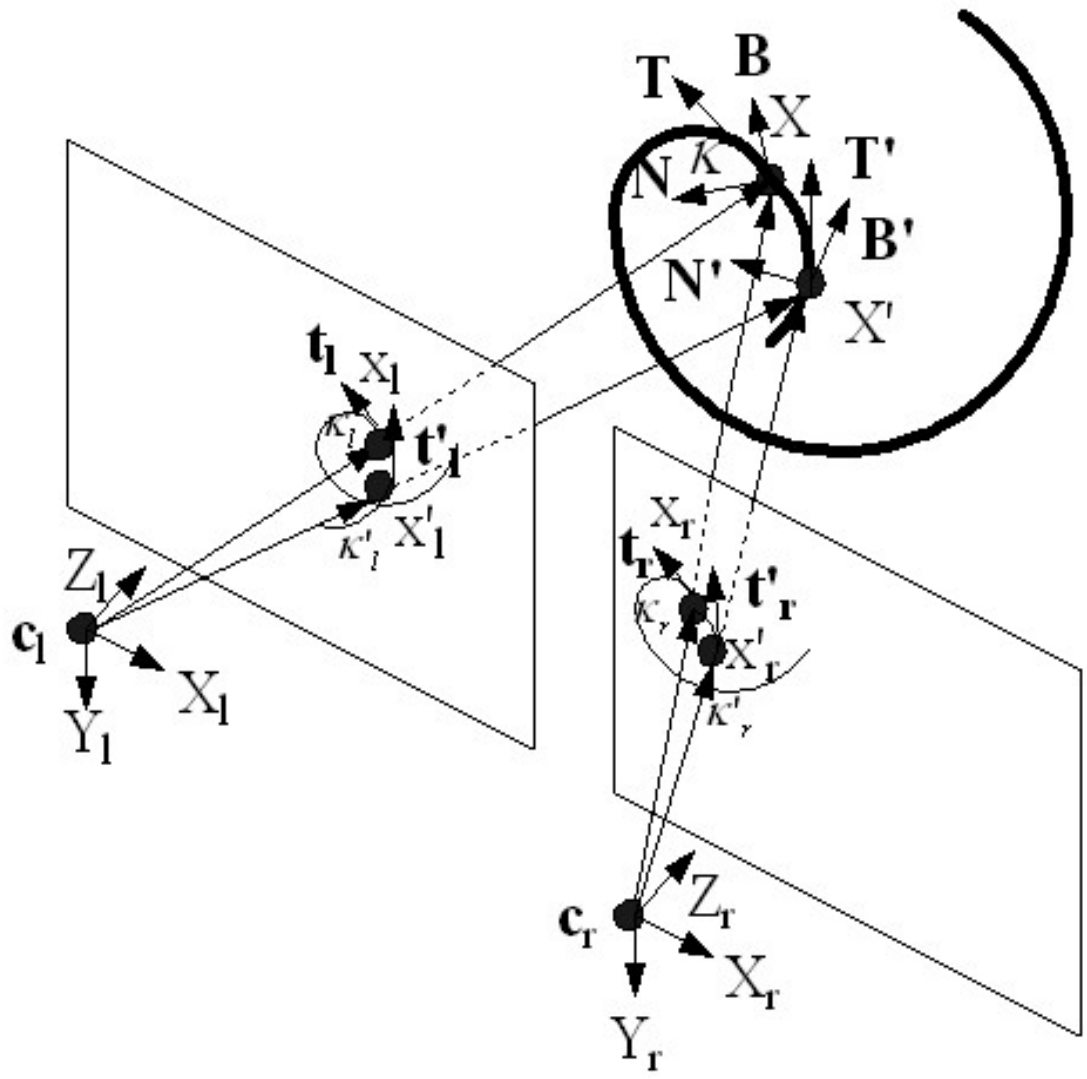


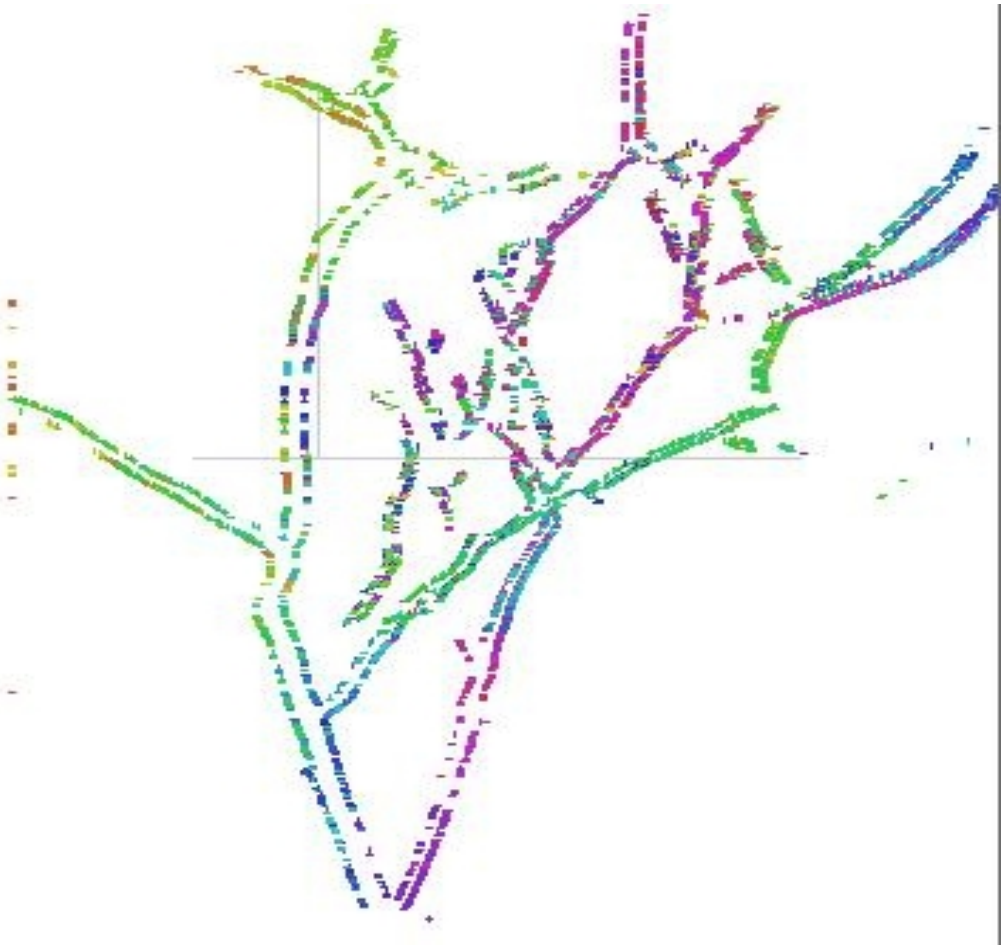
Visual flow coherence in the orientation hypercolumn ($XY\theta$) architecture



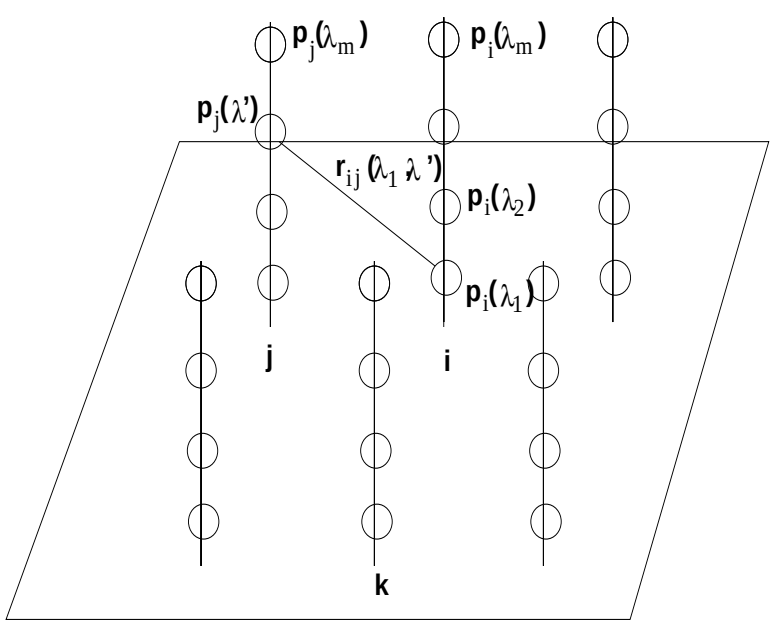
Shading flow field relaxation

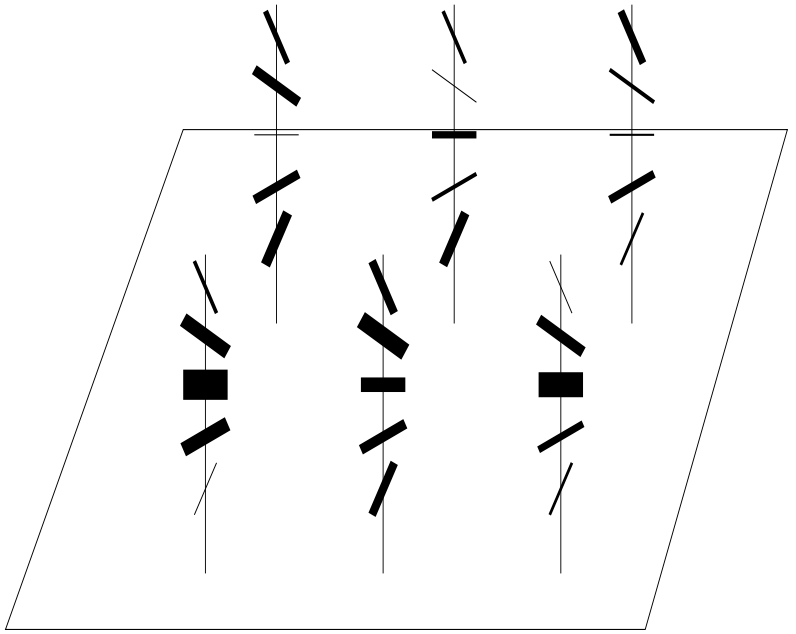






Computational abstraction





"Games Neurons Play"

Consider neurons as "players" in a polymatrix game. Their strategies are whether to fire or not to fire.

Relaxation Labeling	**Game Theory**	**Notation**
nodes	players	$i, j, ..., k$
labels	strategies	λ
probabilistic labeling	mixed strategy	$p_i(\lambda)$
compatibilities	payoff matrix	$r_{i,j}(\lambda, \lambda')$
consistent labeling	Nash equilibrium	Grad $A(P) = 0$

Goal: select labels (strategies) λ at each node (neuron) to extremize:

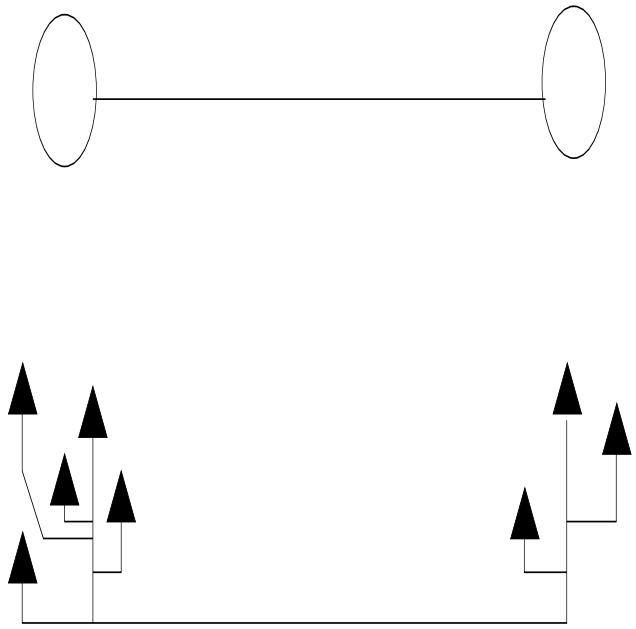
$$A(P) = \sum_{i,\lambda,j,\lambda'} p_i(\lambda) r_{i,j}(\lambda, \lambda') p_j(\lambda')$$

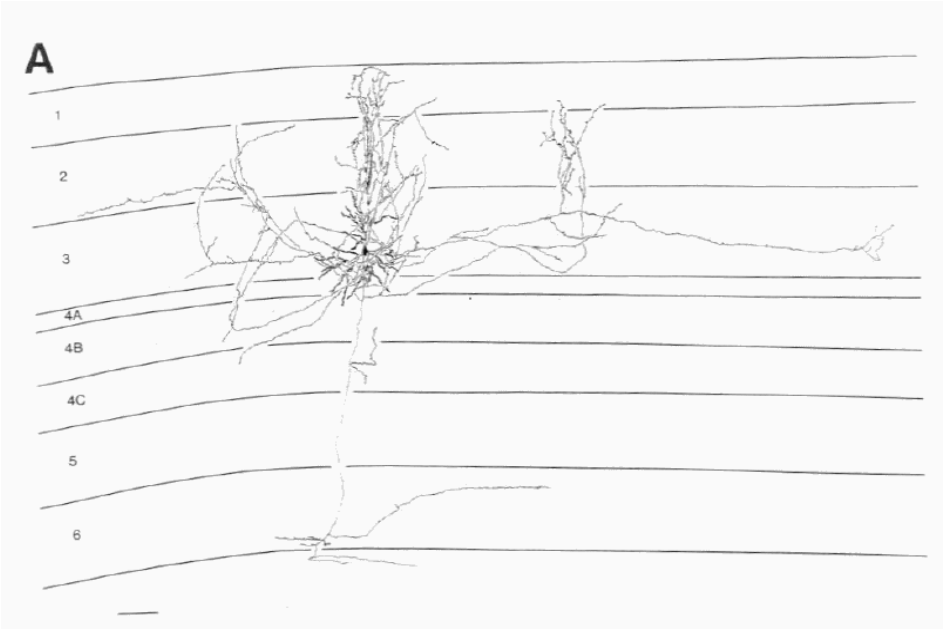
such that

$$\sum_{\lambda} p_i(\lambda) = 1; 0 \leq p_i(\lambda) \leq 1$$

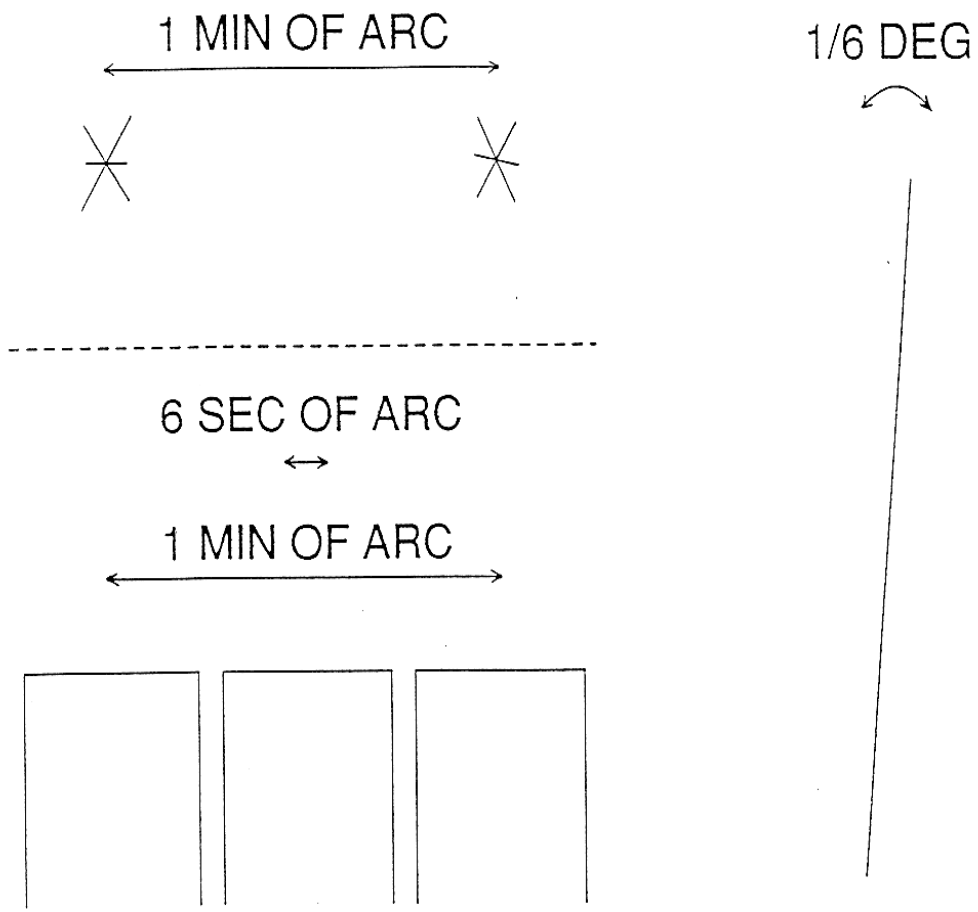
Computational Strategies for LCP's

Computing with cliques of neurons





Acuity and hyperacuity



Hyperaccurate receptive fields and reliability